

Producer Theory Practice Problems

Intermediate Micro

Chapter 7— Comparative Statics

1. Use the information from an earlier problem that we did in class. You have the optimal combinations of capital and labor and the level of output in your notes.

$$Q = 5K^{\frac{1}{2}}L^{\frac{1}{2}}$$

$$MP_L = \frac{5}{2}K^{\frac{1}{2}}L^{-\frac{1}{2}}$$

$$MP_K = \frac{5}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}$$

$$w = \$5$$

$$r = \$20$$

$$TC = \$1,000$$

Suppose that w increased to \$10, r remained at \$20, and total cost was unchanged at \$1,000. Find the optimal quantities of labor and capital that the firm should hire. How much output can the firm produce at the new prices?

2. Use the production function specified in problem #1. Suppose that w increased to \$20, r remained at \$20, and total output was unchanged at 250 units. Find the optimal quantities of labor and capital that the firm should hire. What is the total cost of producing 250 units of output at the new prices?
3. Suppose that a particular machine requires two workers to produce two units of output. Thus, the production function is $Q = \min \{2K, L\}$.

$$TC = \$1,000$$

$$w = \$10$$

$$r = \$20$$

Find the optimal quantities of labor and capital that the firm should hire. How much output can the firm produce?

Suppose that r decreased to \$5, w remained at \$10, and total cost was unchanged at \$1,000. Find the optimal quantities of labor and capital that the firm should hire. How much output can the firm produce at the new prices?

4. Use the production function specified in problem #3. Suppose that r decreased to \$5, w remained at \$10, and total output was unchanged at 50 units. Find the optimal quantities of labor and capital that the firm should hire. What is the total cost of producing 50 units of output at the new prices? *What is unique about this problem?*

5. Suppose that in order to produce one unit of output a firm can use either one hour of labor time or $\frac{1}{4}$ hour of machinery time. Thus, the production function is $Q = L + 4K$.

$$w = \$10$$

$$r = \$20$$

$$TC = \$800$$

Find the optimal quantities of labor and capital that the firm should hire. How much output can the firm produce?

Suppose that r increased to \$44, w remained at \$10, and total cost was unchanged at \$800. Find the optimal quantities of labor and capital that the firm should hire. How much output can the firm produce at the new prices?

6. Use the production function specified in problem #5. Suppose that r increased to \$44, w remained at \$10, and total output was unchanged at 160 units. Find the optimal quantities of labor and capital that the firm should hire. What is the total cost of producing 160 units of output at the new prices?

Chapter 7 — Comparative Statics
Cobb-Douglas Production Functions

7. $Q=K^2L$
 $MP_L=K^2$
 $MP_K=2KL$

$w=\$8$
 $r=\$10$
 $TC=\$800$

A. Find K^* , L^* , and Q^* .

For parts B through E, refer to the above setup. Change only the one variable listed.

B. Now suppose that $TC=\$400$. Find K^* , L^* , and Q^* .

C. Now suppose that $w=\$4$. Find K^* , L^* , and Q^* .

D. Now suppose that $r=\$20$. Find K^* , L^* , and Q^* .

E. Now suppose that $TC=\$1,600$. Find K^* , L^* , and Q^* .

For parts F through H, ignore $TC=\$800$. Instead, the firm is subject to an output constraint. The firm must produce 1,000 units, so $Q=K^2L=1,000$.

F. Find K^* , L^* , and TC^* .

G. Now suppose that $w=\$4$. Find K^* , L^* , and TC^* .

H. Now suppose that $r=\$20$. Everything else remains as in the original specification, with the output constraint of $Q=K^2L=1,000$. Find K^* , L^* , and TC^* .

Chapter 8 — Costs

8. $Q = K^{1/2}L^{1/2}$
 $MP_L = 1/2 K^{1/2} L^{-1/2}$
 $MP_K = 1/2 K^{-1/2} L^{1/2}$
 $w = \$5$
 $r = \$10$
- A. Derive the equation for the total cost curve.
 - B. Find the LRAC (long-run average cost curve) and identify the economies of scale.
 - C. Does your answer to part B match with the returns to scale of the production function?
9. $TC = 1,000Q - 30Q^2 + Q^3$
 $MC = 1,000 - 60Q + 3Q^2$
- A. Find the price and output level that minimizes ATC.
 - B. Find the price and output level that minimizes AVC.
10. $TC = 1,000 + 50Q^2$
 $MC = 100Q$
- A. Find the price and output level that minimizes ATC.
 - B. Find the price and output level that minimizes AVC.

Answers to Producer Theory Practice Problems

1. $L^{**}=50, K^{**}=25, Q_2=176.75$, so L^{**} decreases and Q decreases
2. $L^{***}=50, K^{***}=50, TC_2=\$2,000$, so L^{***} decreases, K^{***} increases, and TC increases
3. $L^*=50, K^*=25, Q_1=50$; then $L^{**}=80, K^{**}=40, Q_2=80$, so L^{**} increases, K^{**} increases, and Q increases
4. $L^{***}=50, K^{***}=25, TC_2=\625 ; note that L^{***} and K^{***} cannot change but TC decreases
5. $L^*=0, K^*=40, Q_1=160$; then $L^{**}=80, K^{**}=0, Q_2=80$, so firm switches from only K to only L and Q decreases
6. $L^{***}=160, K^{***}=0, TC_2=\$1,600$, so TC increases

Cobb-Douglas Optimization #7

- A. $K^*=53.33, L^*=33.33, Q^*=94,793.48$
- B. $K^*=26.67, L^*=16.67, Q^*=11,857.19$
- C. $K^*=53.33, L^*=66.67, Q^*=189,615.4$
- D. $K^*=26.67, L^*=33.33, Q^*=23,707.26$
- E. $K^*=106.67, L^*=66.67, Q^*=758,603.85$
- F. $K^*=11.70, L^*=7.31, TC^*=\175.44
- G. $K^*=9.28, L^*=11.6, TC^*=\139.20
- H. $K^*=9.28, L^*=11.6, TC^*=\278.40

Chapter 8 Cost Problems

8.
 - A. $TC=14.14Q$
 - B. $ATC=TC/Q=14.14$, which implies minimum efficient scale
 - C. The cost average cost function is in the minimum efficient scale region, which implies that the production function exhibits constant returns to scale. Is this true? Yes, $Q=K^{1/2}L^{1/2}$ exhibits CRS.
9.
 - A. $P=\$775, Q=15$
 - B. same answer as part A because there are no fixed costs
10.
 - A. $P=\$447, Q=4.47$
 - B. $P=\$0, Q=0$