

Managerial Economics
Oligopoly and Prisoners' Dilemma Practice Problems

Cournot Duopoly Problem:

Suppose that two duopolists face the demand curve $P = 100 - Q$. Each firm has an identical total cost function of $TC = 10Q$. Suppose that each firm follows a Cournot strategy. Find the market price, outputs for each firm, and profits for each firm.

Stackelberg Price Leader Problem:

Now suppose that Firm B commits to the Cournot strategy. Show how Firm A can use this information to its advantage. Find the market price, outputs for each firm, and profits for each firm.

Prisoners' Dilemma Problem:

Suppose that two firms, A and B, are considering introducing new products. Each firm's monthly profits (in thousands of dollars) are listed in the table below:

		Company B		
		Product 4	Product 5	Product 6
<i>Company A</i>	<i>Product 1</i>	<i>1, 5</i>	<i>2, 2</i>	<i>3, 6</i>
	<i>Product 2</i>	<i>2, 3</i>	<i>3, 5</i>	<i>4, 8</i>
	<i>Product 3</i>	<i>4, 6</i>	<i>5, 8</i>	<i>3, 7</i>

Is there a dominant strategy for either player?

Is there a Nash equilibrium?

Cournot Answer:

$$P = 100 - (Q_A + Q_B)$$

$$TC_A = 10Q_A$$

$$TC_B = 10Q_B$$

$$\Pi_A = TR - TC = P \times Q - TC = (100 - Q_A - Q_B)Q_A - 10Q_A$$

$$\Pi_A = 100Q_A - Q_A^2 - Q_AQ_B - 10Q_A = 90Q_A - Q_A^2 - Q_AQ_B$$

$$\frac{d\Pi_A}{dQ_A} = 90 - 2Q_A - Q_B$$

$$dQ_A$$

$$\text{set} = 0$$

$$90 - 2Q_A - Q_B = 0$$

$$90 - Q_B = 2Q_A$$

$$Q_A = 45 - 0.5Q_B$$

Because the cost functions are identical, the reaction functions are similar.

$$Q_B = 45 - 0.5Q_A$$

$$Q_A = 45 - 0.5(45 - 0.5Q_B) = 45 - 22.5 + 0.25Q_A$$

$$Q_A - 0.25Q_A = 22.5$$

$$0.75Q_A = 22.5$$

$$Q_A = 30 \quad \Pi_A = 90Q_A - Q_A^2 - Q_AQ_B = 2700 - 900 - 900 = \$900$$

$$Q_B = 30 \quad \Pi_B = \$900$$

$$P = \$40$$

Stackelberg Answer:

Suppose firm B commits to its Cournot strategy and follows the reaction function

$$Q_B = 45 - 0.5Q_A.$$

Firm A uses this information to maximize its profits.

$$\Pi_A = TR - TC = P \times Q - TC = (100 - Q_A - Q_B)Q_A - 10Q_A$$

$$\Pi_A = 100Q_A - Q_A^2 - Q_AQ_B - 10Q_A$$

$$\Pi_A = 100Q_A - Q_A^2 - Q_A(45 - 0.5Q_A) - 10Q_A = 90Q_A - Q_A^2 - 45Q_A + 0.5Q_A^2$$

$$\Pi_A = 45Q_A - 0.5Q_A^2$$

$$\frac{d\Pi_A}{dQ_A} = 45 - Q_A$$

$$dQ_A$$

$$\text{set} = 0$$

$$Q_A = 45 \quad \Pi_A = 90Q_A - Q_A^2 - Q_AQ_B = 4050 - 2025 - 1012.5 = \$1,012.50$$

$$Q_B = 22.5 \quad \Pi_B = 90Q_B - Q_B^2 - Q_AQ_B = 2025 - 506.25 - 1012.5 = \$506.25$$

$$P = \$32.50$$

Does this make sense? Firm A makes a higher profit than under the Cournot strategy and does so at the expense of Firm B's lower profits.

Prisoners' Dilemma Answer:

Suppose that two firms, A and B, are considering introducing new products. Each firm's monthly profits (in thousands of dollars) are listed in the table below:

		Company B		
		Product 4	Product 5	Product 6
Company A	Product 1	1, 5	2, 2	3, 6
	Product 2	2, 3	3, 5	4, 8
	Product 3	4, 6	5, 8	3, 7

If Company A introduces product 1, Company B should introduce product 6.
If Company A introduces product 2, Company B should introduce product 6.
If Company A introduces product 3, Company B should introduce product 5.
Company B does NOT have a dominant strategy.

If Company B introduces product 4, Company A should introduce product 3.
If Company B introduces product 5, Company A should introduce product 3.
If Company B introduces product 6, Company A should introduce product 2.
Company A does NOT have a dominant strategy.

Notice that regardless of what Company A does, Company B never introduces product 4.
Notice that regardless of what Company B does, Company A never introduces product 1.
Each player has a **dominated strategy**, meaning a strategy that is always inferior.
Let's remove Product 1 and Product 4.

		Company B	
		Product 5	Product 6
Company A	Product 2	3, 5	4, 8
	Product 3	5, 8	3, 7

If Company A introduces product 2, Company B should introduce product 6.
If Company A introduces product 3, Company B should introduce product 5.
Company B does NOT have a dominant strategy.

If Company B introduces product 5, Company A should introduce product 3.
If Company B introduces product 6, Company A should introduce product 2.
Company A does NOT have a dominant strategy.

There is no dominant strategy for either player.
There is no Nash equilibrium.